

Question Paper Code: 42764

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

First Semester
Civil Engineering
MA 2111 - MATHEMATICS - I
(Common to all Branches)
(Regulations 2008)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

PART - A

 $(10\times2=20 \text{ Marks})$

- 1. Find the sum and product of the Eigen values of the matrix $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.
- 2. Use Cayley-Hamilton theorem to find the inverse of the matrix $A = \begin{pmatrix} 7 & 3 \\ 2 & 6 \end{pmatrix}$.
- 3. Find the equation of the sphere whose centre is (1, 2, 3) and which touches the plane 2x + 2y z = 2.
- 4. Find the equation of the right circular cone whose vertex is the origin and axis is the positive z-axis.
- 5. Define Radius of curvature of a curve.
- 6. Find the envelope of the family of lines $y = mx + \frac{a}{m}$, m being a parameter.
- 7. If $u = xy \log (xy)$, express du in terms of dx and dy.
- 8. State any two properties of Jacobians.



- 9. Find the limits of integration in the double integral $\iint f(x,y) dxdy$, where R is in the first quadrant and bounded by x = 0, y = x and y = 1.
- 10. Sketch roughly the region of integration for the integral $\int_0^b \int_0^{\frac{a}{b}(b-y)} f(x,y) dx dy.$

 $(5\times16=80 \text{ Marks})$

11. a) i) Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$
 (6)

ii) Verify if the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$ satisfies its own characteristic

equation. If so, find its inverse. (10)

b) Find the canonical form of the quadratic expression:

$$2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_1x_3.$$
 (16)

- 12. a) i) Find the equation of the cone whose vertex is (3, 1, 2) and the base curve is $2x^2 + 3y^2 = 1$ and z = 1.
 - ii) Find the equation of the sphere passing through the circle given by $x^2 + y^2 + z^2 + 3x + y + 4z 3 = 0$; $x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$ and the point (1, -2, 3).

(OR)

- b) i) Find the equation of the right circular cylinder of radius 3 units whose axis is the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$. (8)
 - ii) Find the equation of sphere having the circle. $x^2 + y^2 + z^2 + 10y 4z 8 = 0, x + y + z = 3 \text{ as a great circle.}$ (8)

(6)



- 13. a) i) Find the equation of the circle of curvature of the curve $x^3 + y^3 = 3$ axy at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (10)
 - ii) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters "a" and "b" are connected by the relation $ab = c^2$. (6)
 - b) i) Find the radius of curvature at (a, 0) on the curve $xy^2 = a^3 x^3$. (6)
 - ii) Find the evolutes of the hyperbola $\frac{x^2}{a^2} \frac{y^2}{b^2} = 1$. (10)
- 14. a) i) Expand $xy^2 + 2x 3y$ in powers of (x + 2) and (y 1) upto the third degree terms. (8)
 - ii) Examine the function $x^3 + y^3 = 3axy$ for its extreme values. (8)
 (OR)
 - b) i) Examine the functional dependence of the functions $u = \frac{x+y}{x-y}$ and $v = \frac{xy}{(x-y)^2}$. If they are dependent, find the relation between them.
 - ii) A rectangular box, open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction. (10)
- 15. a) i) Change the order of integration in $\int_0^{2a} \int_{\frac{x^2}{4a}}^{a} (x+y) dx dy$ and then evaluate it. (8)
 - ii) Evaluate $\iiint \sqrt{1-x^2-y^2-z^2} dx dy dz$, where V is taken through the volume of the sphere $x^2+y^2+z^2=1$. (8)
 - b) i) Transform the double integral $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \frac{x \, dx dy}{\sqrt{x^2+y^2}}$ in polar coordinates and then evaluate it. (8)
 - ii) Evaluate $\int_{0}^{\log 2} \int_{0}^{x} \int_{0}^{x+y} e^{x+y+z} dxdydz$ (8)

If you is a found the equipment the street of convenience of the curve x a x y = 1 any at

this point $\frac{1}{2} \cdot \frac{1}{2}$ into quite

- in front that constage of the finite of those $\frac{1}{2} = \frac{1}{2} = 1$, where the parameters of the continue of
- (0) "x "n = "gr ocurs sit medit a) to sufficient to solder air baril (
- $I = \frac{r_{\rm eff}}{r_{\rm eff}} \cdot \frac{r_{\rm eff}}{r_{\rm eff}} = 1$ (101)
- to at a Sepand spirit in powers of or a limit (c. 1) upon the third degree of the standard degree (a)
- in Examine the function $y^2 + y^2 = 3 \exp \int_{\mathbb{R}^2} \ln x \cos x \sin x \cos x$ (6)
 - for it fixeasing the functional dependence of the functions $u = \frac{1}{x-y}$ and
- $v = \frac{2iN}{(n-v)^2}$ If there they depend out, find the relation between them. (0)
- in A restanguidar han, opin in the top, is to have a voltage of 12 on Find the dimension of the loss, that magnitud the hand muturial for its fourtweetiers. (10)
- (B) it is the contract the contract of the first product of the pr
- on a valuable of the splane of $x = y^2 x^2$ during a valuable of the splane of the
 - in it Transform the double integral [. [x dody in point coordinates and
- (in exclusive and r